The Interaction of Radiation with Matter: Theory and Practice

William L. Dunn[1] and Richard P. Hugtenburg[2,3]

[1]Mechanical and Nuclear Engineering Department, Kansas State University
[2]School of Physics and Astronomy, University of Birmingham
[3]Queen Elizabeth Medical Centre, University Hospital Birmingham

IEEE Short Course

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Preview

- Session 1, Overview and photon interactions
  - Ionizing radiation and phase space and field quantities
  - Photon interactions
- Session 2, Charged particle interactions
  - Electron interactions
  - Heavy charged particle interactions
- Session 3, Neutron interactions
- Session 4, Radiation detection and applications

Session 1

- Ionizing radiation
- Phase space and field quantities
  - Flux density and fluence
  - Current density and flow
  - Cross sections
  - Reaction rate density and reaction rate
- Photon interactions
  - Photoelectric absorption
  - Compton scattering
  - Pair production
  - Other special cases
- Data resources

Ionizing Radiation

- Radiation encompasses a vast array of particle/wave types (cosmic radiation, X rays, protons, electrons, alpha particles, visible light, microwaves, radio waves, etc.)
- We will focus on radiation that directly or indirectly ionizes as it traverses matter
- Ionization is separation of electrons from stable nuclei into electron-ion pairs
- Thus, we limit ourselves to
  - X and γ rays
  - charged particles (protons, deuterons, alpha particles, etc.)
  - neutrons (which ionize indirectly)

Phase Space

- We should first understand the independent variables with which we must deal; these include
  - Position **r**, generally expressed in rectangular (x, y, z), cylindrical (ρ, θ, z), or spherical (ρ, θ, ψ) coordinates
  - Energy E, with units expressed in eV, keV, or MeV
  - Direction $\mathbf{\Omega}$, which can be expressed in terms two angles
  - Time t
- These form a seven-dimensional "phase space"

$$\mathbf{P} = \left(\mathbf{r}, E, \mathbf{\Omega}, t\right)$$

Direction Variables

- We take the 3-D velocity vector **v** and express it in terms of energy *E*, which is related to speed or wavelength (wave-particle duality), and direction **Ω**, which can be expressed as

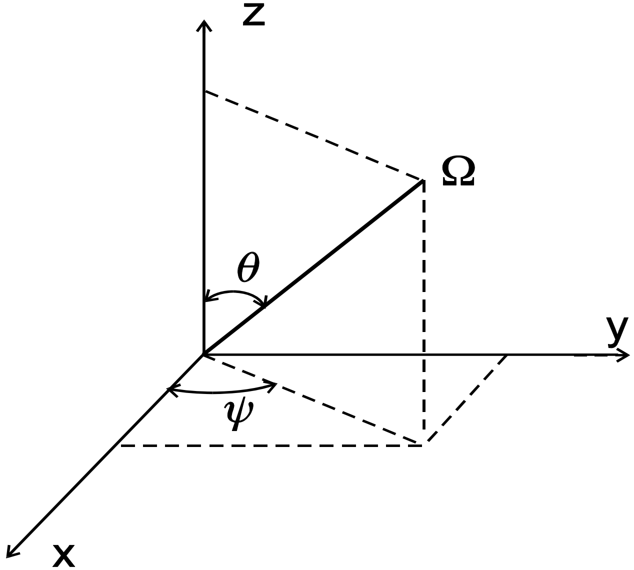
$$\mathbf{\Omega} = \left(\sin\theta\cos\psi, \sin\theta\sin\psi, \cos\theta\right)$$

or equivalently as

$$\mathbf{\Omega} = \left(\sqrt{1-\omega^2}\cos\psi, \sqrt{1-\omega^2}\sin\psi, \omega\right)$$

where

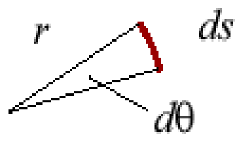
$$\omega = \cos\theta$$



Differential Plane Angle

- The differential arc length *ds* subtended by a differential angle $d\theta$ using polar coordinates is

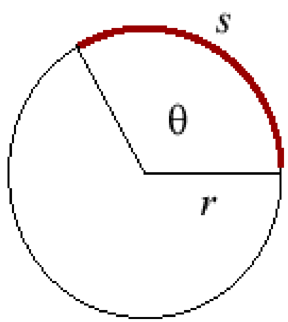
$$ds = r d\theta$$



- Integrating, we see

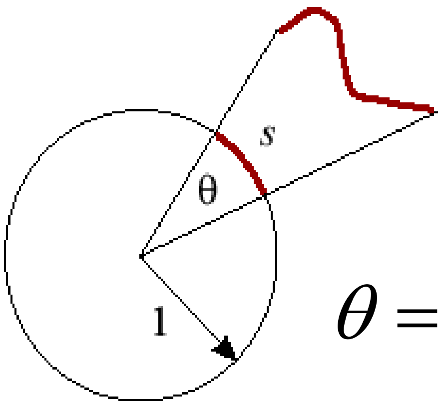
$$s = \int_0^{\theta} r d\theta' = r\theta$$

and thus $$\theta = \frac{s}{r}$$

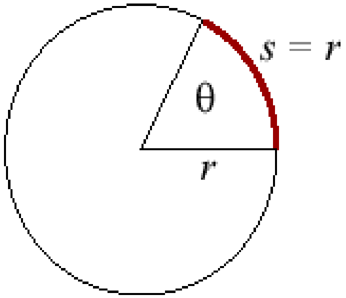


Plane Angle

- Thus, plane angle can be thought of as *the length of an arc that a line in a plane projects onto the unit circle*
- An angle of 1 radian (rad) is the angle that is swept out on a circle of radius *r* by an arc equal to the circle radius
- A circle has 2π rad



$$\theta = \frac{s}{1} = s$$



$$\theta = \frac{s}{r} = \frac{r}{r} = 1 \text{ rad}$$

Differential Solid Angle

- The differential area dA on the surface of a sphere can be expressed in spherical coordinates as

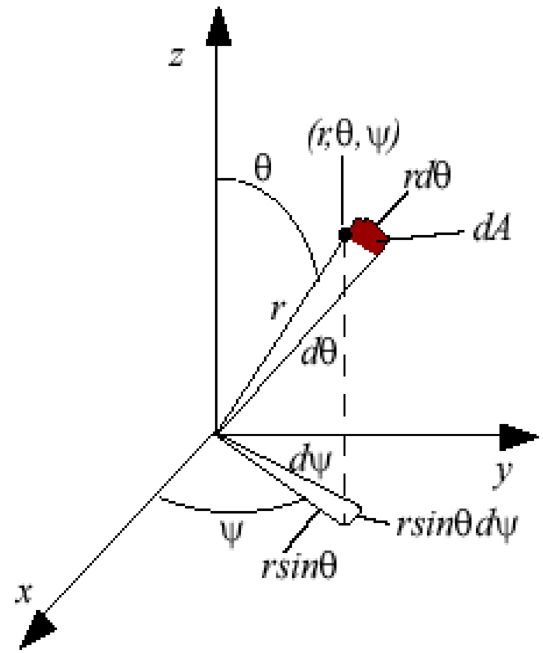
$$dA = r^2 \sin\theta d\theta d\psi$$

- The differential solid angle is the differential area on a unit sphere

$$d\Omega = \sin\theta d\theta d\psi$$

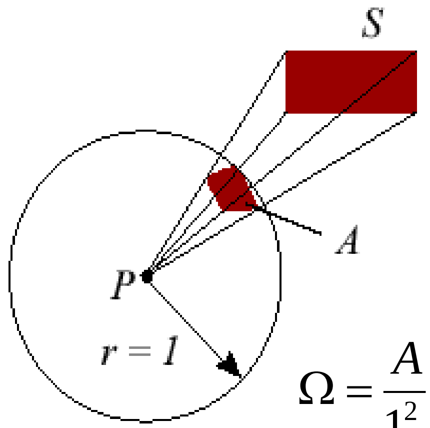
- Integrating, we see

$$A = \int_0^{\psi} \int_0^{\theta} r^2 \sin\theta' d\theta' d\psi' = r^2\Omega \Rightarrow \Omega = \frac{A}{r^2}$$

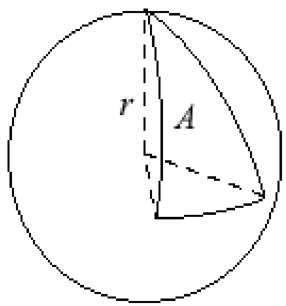


Solid Angle

- The solid angle subtended by a surface *S* at point *P* thus can be thought of as the area that *S* projects onto the unit sphere centered at *P*
- Then, 1 steradian (sr) is the solid angle that subtends on a sphere of radius *r* an area (of any shape) equal to r^2



$$\Omega = \frac{A}{1^2} = A$$



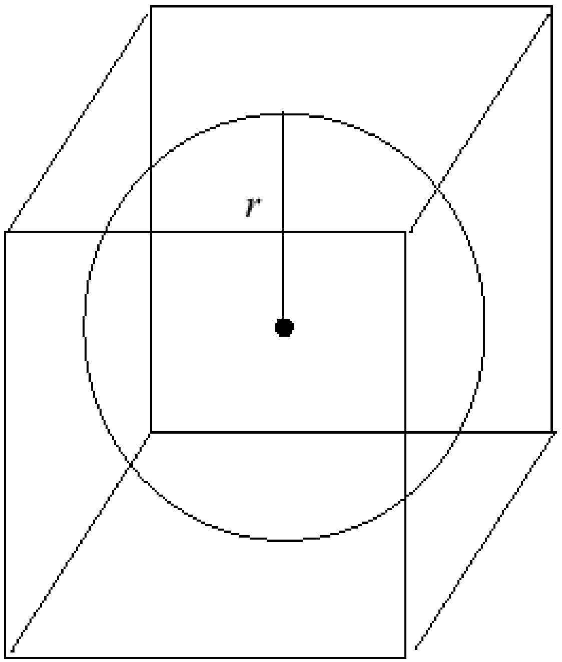
$$\Omega = \frac{A}{r^2} = \frac{r^2}{r^2} = 1 \text{ sr}$$

Solid Angle

- Solid angle is a ratio of areas

$$\Omega = \frac{A}{r^2}$$ where A is the area projected onto a sphere of radius r

- Since the surface area of a sphere is $4\pi r^2$, any surface completely surrounding a point subtends 4π sr



Field Quantities

- The primary dependent variables with which we will be concerned include
  - Radiation density
  - Flux density (often called flux) and fluence
  - Current density (also called current or flow vector rate) and flow vector
  - Reaction rate density or reaction rate
- The current density is a vector quantity analogous to heat rate in heat transfer; density, flux density, and reaction rate density are scalar quantities

Radiation Density

- Density is the number of particles per unit volume
- But not all particles have the same speed or direction, so we *define* the energy- and direction-dependent density as follows

$n(\mathbf{r}, E, \mathbf{\Omega}, t)\, d^3r dE d\Omega =$ expected number of particles within d^3r about $\mathbf{r}$
having energies within dE about E and
directions within $d\mathbf{\Omega}$ about $\mathbf{\Omega}$ at time t

$d^3r = dxdydz$	rectangular
$d^3r = \rho d\rho d\theta dz$	cylindrical
$d^3r = \rho^2 \sin\theta d\rho d\theta d\psi$	spherical

Units of $n(\mathbf{r}, E, \mathbf{\Omega}, t)$ are cm^{-3} MeV^{-1} sr^{-1}

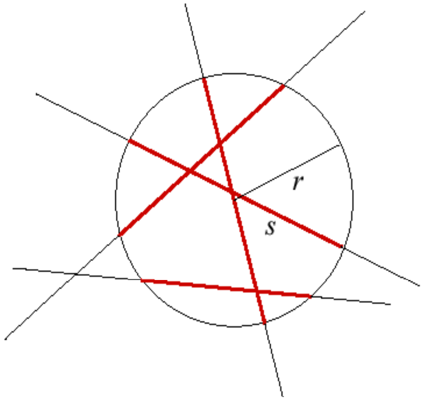
Flux Density

- We define the energy- and direction-dependent flux density to be $\varphi(\mathbf{r},E,\mathbf{\Omega},t)=v(E)n(\mathbf{r},E,\mathbf{\Omega},t)$
- The total flux density can be written

$$\varphi(\mathbf{r},t)=\int_0^\infty\int_{4\pi}\varphi(\mathbf{r},E,\mathbf{\Omega},t)\,d\Omega dE$$

or expressed as

$$\varphi(\mathbf{r},t)=\lim_{r\to 0}\lim_{\Delta t\to 0}\frac{\sum_{i=1}^{N}s_i}{\frac{4}{3}\pi r^3\Delta t}$$



Fluence

- Related to flux density is the fluence, which can be expressed as

$$\Phi(\mathbf{r}, E, \mathbf{\Omega}, t) = \int_{t_0}^{t} \varphi(\mathbf{r}, E, \mathbf{\Omega}, t')\, dt' \quad \left\{ \text{cm}^{-2}\, \text{MeV}^{-1}\, \text{sr}^{-1} \right\}$$

- The fluence is the flux density accumulated over some time interval
- Fluence at time t depends on the starting time t_0
- Flux density can be seen to be

$$\varphi(\mathbf{r}, E, \mathbf{\Omega}, t) = \frac{\partial \Phi(\mathbf{r}, E, \mathbf{\Omega}, t)}{\partial t}$$

Current Density Vector

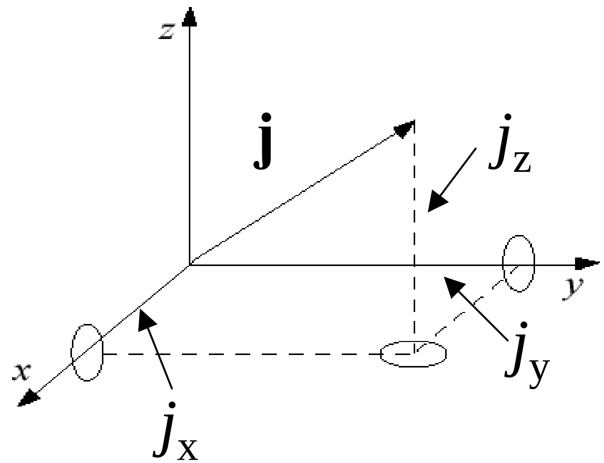
- Flux density is a scalar quantity
- If we multiply the 7-phase-space neutron density by the velocity vector, we get the energy- and direction-dependent *current density vector*

$$
\begin{aligned}
\mathbf{j}(\mathbf{r},E,\mathbf{\Omega},t) &\triangleq \mathbf{v}n(\mathbf{r},E,\mathbf{\Omega},t) \\
&= \mathbf{\Omega}\varphi(\mathbf{r},E,\mathbf{\Omega},t) \\
&= j_x(\mathbf{r},E,\mathbf{\Omega},t)\mathbf{i}_x + j_y(\mathbf{r},E,\mathbf{\Omega},t)\mathbf{i}_y + j_z(\mathbf{r},E,\mathbf{\Omega},t)\mathbf{i}_z
\end{aligned}
$$

Units are $\mathrm{cm}^{-2}\ \mathrm{MeV}^{-1}\ \mathrm{sr}^{-1}\ \mathrm{s}^{-1}$

Current Density Vector Components

- $j_x(\mathbf{r}, E, \mathbf{\Omega})\,dE\,d\mathbf{\Omega}$ is the rate at which particles having energies within dE about E and moving in directions within $d\mathbf{\Omega}$ about $\mathbf{\Omega}$ cross a unit area at $\mathbf{r}$ perpendicular to the x-axis
- Similar definitions for $j_y(\mathbf{r}, E, \mathbf{\Omega}, t)$ and $j_z(\mathbf{r}, E, \mathbf{\Omega}, t)$

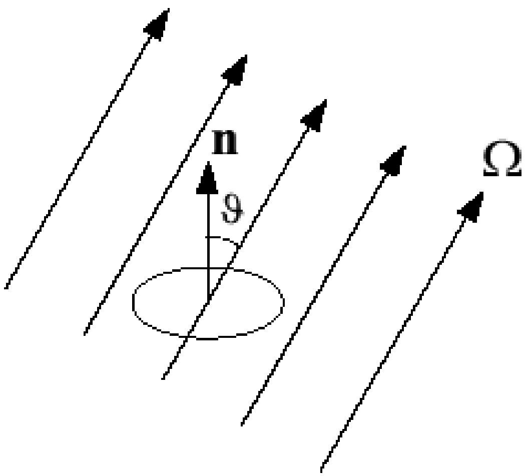


Angular Current Density Component

- Define $j_n(\mathbf{r},\mathbf{\Omega}) \triangleq \mathbf{n}\bullet\mathbf{j}(\mathbf{r},\mathbf{\Omega}) = \mathbf{n}\bullet\mathbf{\Omega}\varphi(\mathbf{r},\mathbf{\Omega})$

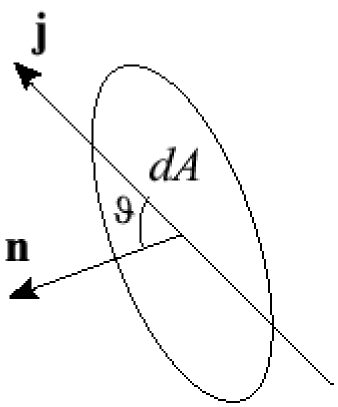
  as the current density in direction $\mathbf{\Omega}$ through a unit area perpendicular to **n**
- **n** can be *x*, *y*, *z*, *r*, or any direction
- But $\mathbf{n}\bullet\mathbf{\Omega} = \cos\vartheta$ where ϑ is the angle between **n** and $\mathbf{\Omega}$
- Thus,

$$\boxed{j_n(\mathbf{r},\mathbf{\Omega}) = \cos\vartheta\varphi(\mathbf{r},\mathbf{\Omega})}$$



Total Current Density Component

- In general, for the total current density $\mathbf{j}(\mathbf{r})\bullet\mathbf{n}dA=\varphi(\mathbf{r})\mathbf{\Omega}\bullet\mathbf{n}dA=\varphi(\mathbf{r})\cos\vartheta dA$ is the net rate at which neutrons cross an area *dA* whose normal is **n** in the direction of **n**



Here, ϑ is the angle between **j** and **n**
This holds for any *dA* in any direction
Also, it is positive if more neutrons are crossing in the +**n** direction and negative if more neutrons are crossing in the –**n** direction

Flow Vector

- As fluence is to flux density so too is flow to current density
- We can think in terms of rates or of total quantities
- The flow vector is the integral of the current density vector over some time interval

$$\mathbf{J}\left(\mathbf{r},E,\mathbf{\Omega},t\right)=\int_{t_0}^{t}\mathbf{j}\left(\mathbf{r},E,\mathbf{\Omega},t'\right)dt' \quad \left\{\mathrm{cm}^{-2}\,\mathrm{MeV}^{-1}\,\mathrm{sr}^{-1}\right\}$$

Microscopic Cross Section

- Radiation interacts with matter probabilistically
- Think of radiation as particles or waves and electrons and nuclei as targets
- We use σ to represent the microscopic “cross section,” which can be thought of as an “effective area” per target for an interaction
- Has units of area, cm^2 or b (1 b = 10^{-24} cm^2)

Macroscopic Cross Section

- On a macroscopic scale, we multiply the microscopic cross section by the density of targets
- Thus, the macroscopic cross section is

$$\Sigma = N\sigma$$

- Here, N is the target density (cm^{-3})
- Thus macroscopic cross sections have units of cm^{-1}
- We interpret Σ to be the **probability per unit path length of an interaction**

Another Interpretation

- We can now see the microscopic cross section can be thought to be the probability of an interaction per unit differential path length within a volume of 1 cm^3 containing only one target atom, since

$$\sigma = \frac{\Sigma}{N}$$

Reaction Rate Density

- The basic relation for reaction rate density is

$$F(\mathbf{r}, E, \mathbf{\Omega}, t) = \Sigma(\mathbf{r}, E)\varphi(\mathbf{r}, E, \mathbf{\Omega}, t)$$

- Here $\Sigma(\mathbf{r}, E)$ is the macroscopic cross section and $\varphi(\mathbf{r}, E, \mathbf{\Omega}, t)$ is the energy- and direction-dependent flux density at **r**
- The above equation is central to radiation detection and applications

Physical Interpretation

- The reaction rate density is just the flux density times the macroscopic cross section
- Flux density is really the total distance traveled by all particles within a unit volume per unit time
- Macroscopic cross section is really the probability of an interaction per unit path length
- Thus the product is the number of interactions that are likely to occur within a unit volume per unit time

Reaction Rate

- We can integrate over any volume to form the reaction rate within that volume
- Thus

$$F_V(E,\mathbf{\Omega},t) = \int_V \Sigma(\mathbf{r},E)\varphi(\mathbf{r},E,\mathbf{\Omega},t)\,dV$$

- Also, we can form the total number of reactions over some time interval from

$$F_T(\mathbf{r},E,\mathbf{\Omega}) = \int_0^T \Sigma(\mathbf{r},E)\varphi(\mathbf{r},E,\mathbf{\Omega},t)\,dt = \Sigma(\mathbf{r},E)\Phi(\mathbf{r},E,\mathbf{\Omega},T)$$

Photon Interactions

We now begin to consider the interaction of X rays and gamma rays with matter

X Rays and γ Rays

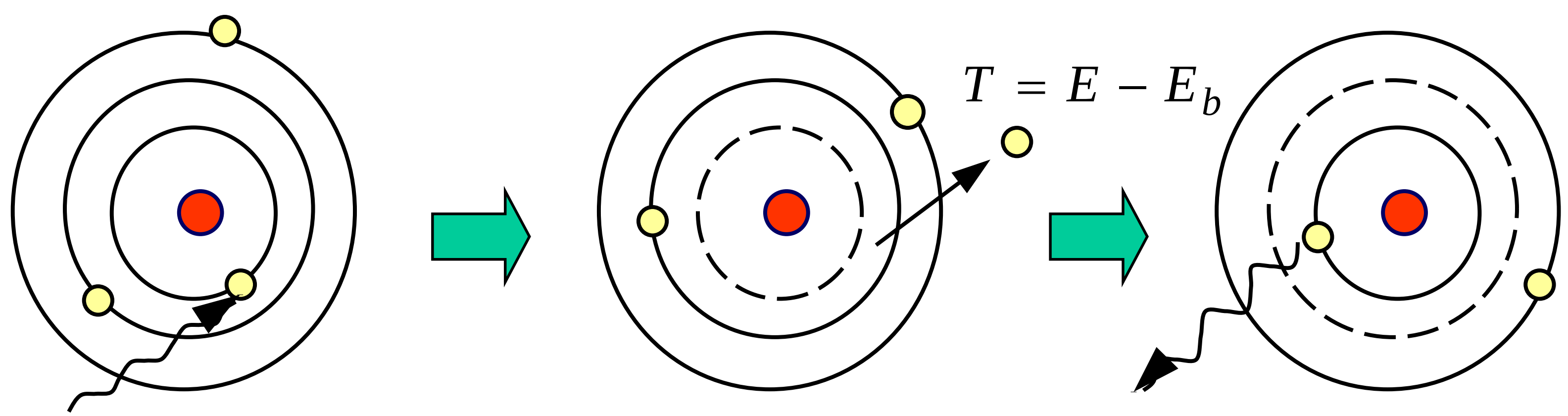
- Both X and γ rays are electromagnetic radiation that can be treated as particles, called photons
- X rays originate in electronic transitions and in electronic radiative losses
- Gamma rays originate in nuclear transitions
- Gamma rays tend to have higher energies than X rays, but this is not always the case
- We often ignore the difference and treat all as ionizing photons
- X rays discovered on 8 November 1895 by Wilhelm Conrad Roentgen

Photon Interactions

- Over the energy range of approximately 1 keV to 100 MeV, photons interact in three main ways
  - Photoelectric absorption
  - Electron scattering
    - Free electron (Klein-Nishina)
    - Bound electron
      - Coherent
      - Incoherent
  - Pair production
- We will consider each of these in some detail
- Then, we will briefly indicate other ways in which ionizing photons interact with matter

Photoelectric Absorption

- The Nobel Prize was awarded to Albert Einstein for one (and only one) discovery, that of photoelectric absorption



$E = h\nu$

Fluorescent X ray or Auger electron cascade

$E_X < E_b$

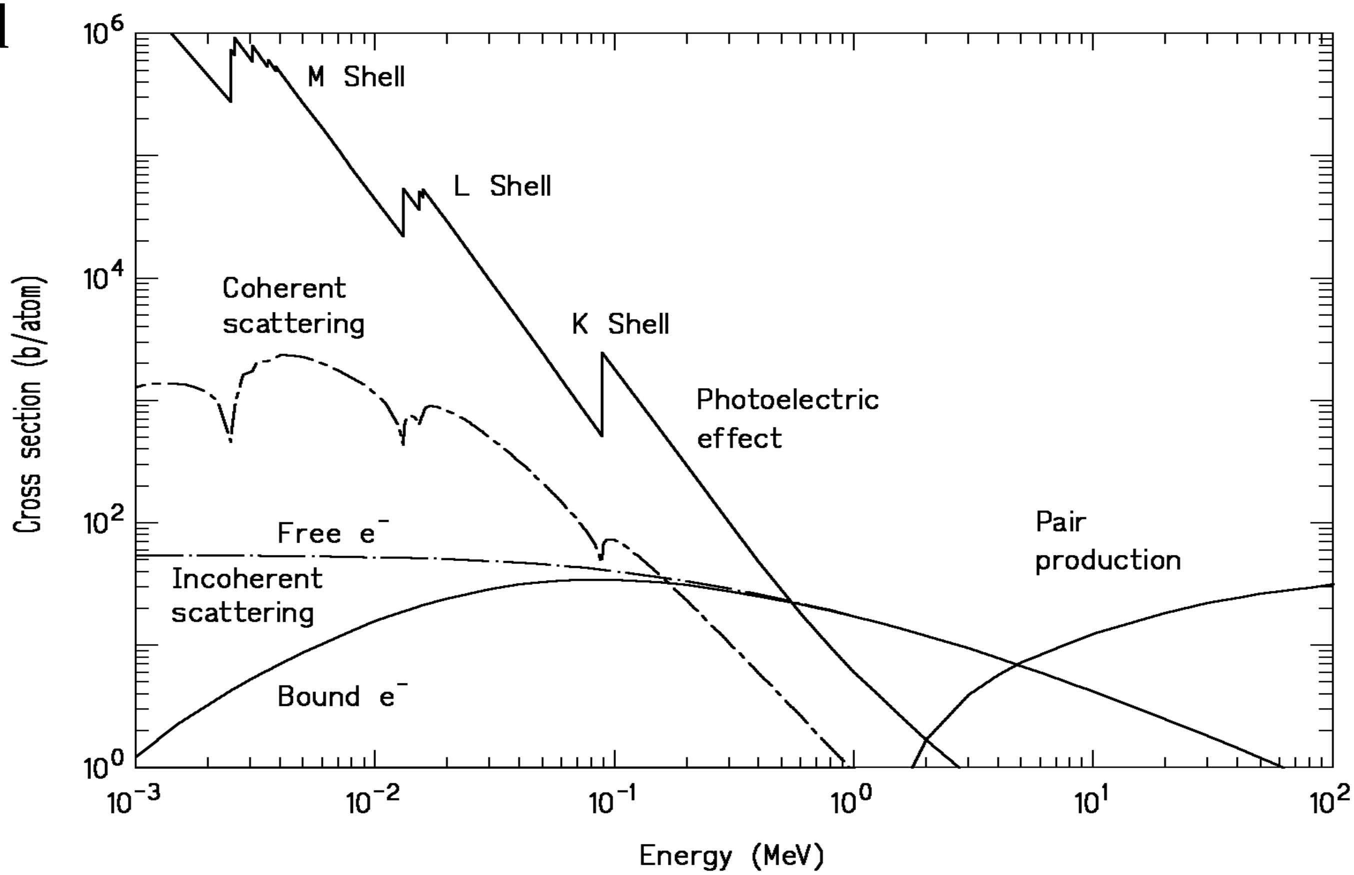
Features of Photoelectric Effect

- $\approx$75% are K-shell interactions
- $\sigma_{ph}(E) \propto Z^4 / E^3$ Low energy phenomenon
- $\omega_K =$ K-shell fluorescence yield ($E_X < E_b$)
- 1-$\omega_K =$ prob. of Auger electron ($E_a \cong E_b$)

Z	ω_K
8	0.005
90	0.965

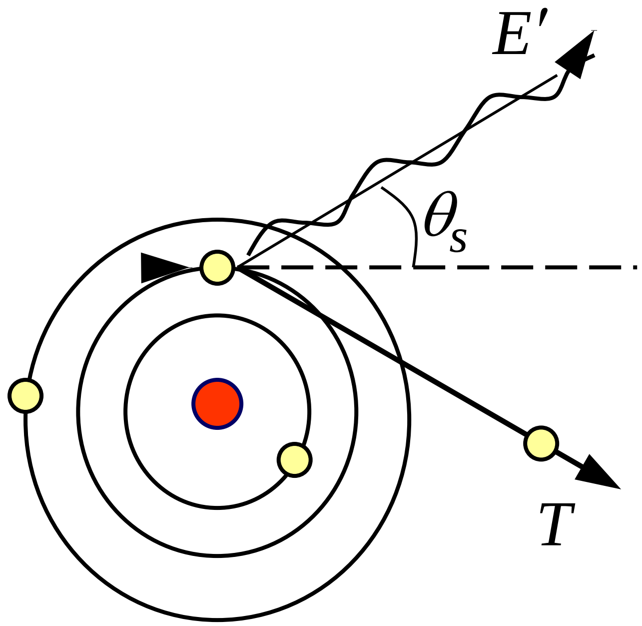
Absorption Edges

Lead

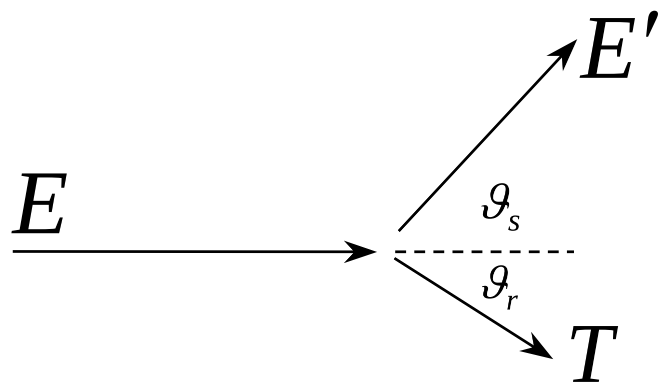


Photon Scattering

- Three basic types
  - Incoherent scattering from free electron (Compton scattering)
  - Incoherent scattering from bound electron
  - Coherent scattering from all atomic electrons in an atom (Rayleigh scattering)



Compton Scattering (Free Electron)



m_e is electron rest mass

$$\boxed{E' = \frac{E}{1 + (E/m_e c^2)\left(1 - \cos\vartheta_s\right)}}, \quad 0 \le \vartheta_s \le \pi$$

$$\lambda' = 1 + \lambda - \cos\vartheta_s \qquad \text{Note: } 0 \le \vartheta_r \le \pi/2$$

Klein-Nishina Formula

Define a reduced energy $\alpha = \frac{E'}{E}$.

The Klein-Nishina formula provides the differential scattering cross section per electron, which can be written as

$$\sigma_{KN}\left(\alpha,\theta_s\right) = \frac{r_e^2}{2}\alpha\left[1+\alpha^2-\alpha\left(1-\cos^2\vartheta_s\right)\right],$$

with r_e the classical electron radius or $r_e = 2.8179\times10^{-13}$ cm.

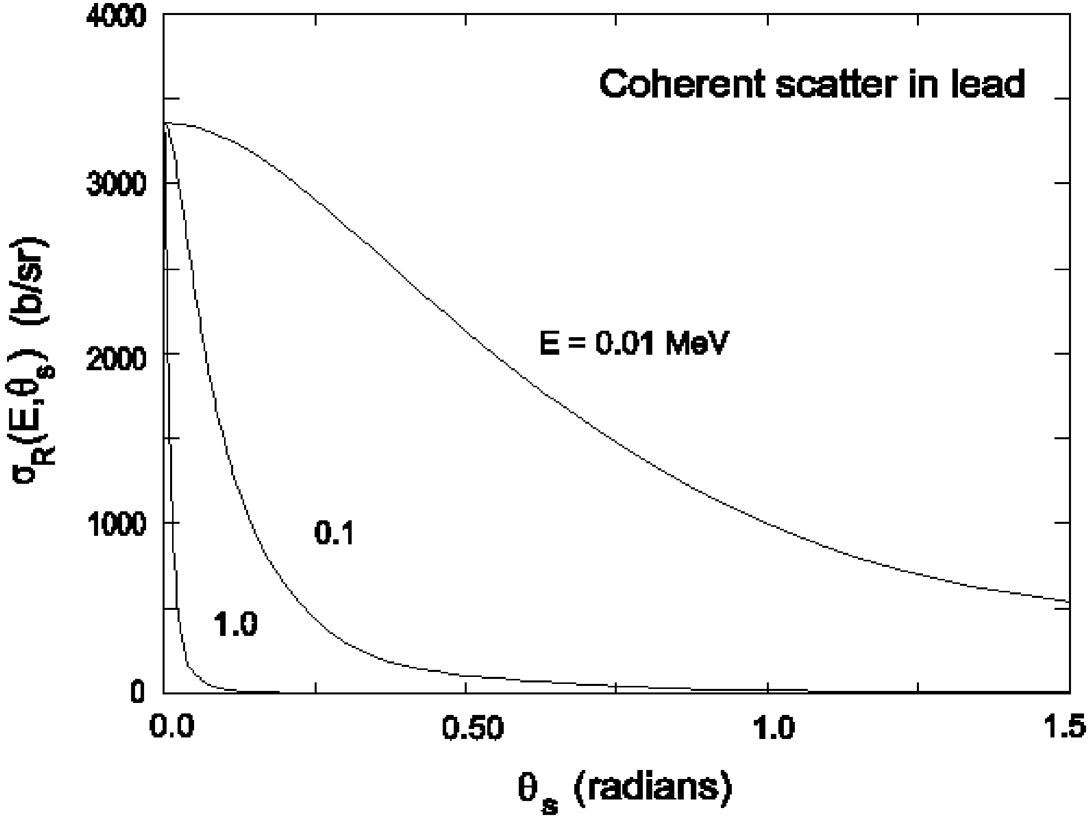
The total Compton cross section per atom is then

$$\sigma_{\mathrm{C}}\left(\alpha\right) = Z\int_0^{\pi}\sigma_{KN}\left(\alpha,\vartheta_s\right)d\vartheta_s\,.$$

Bound Electron Effects

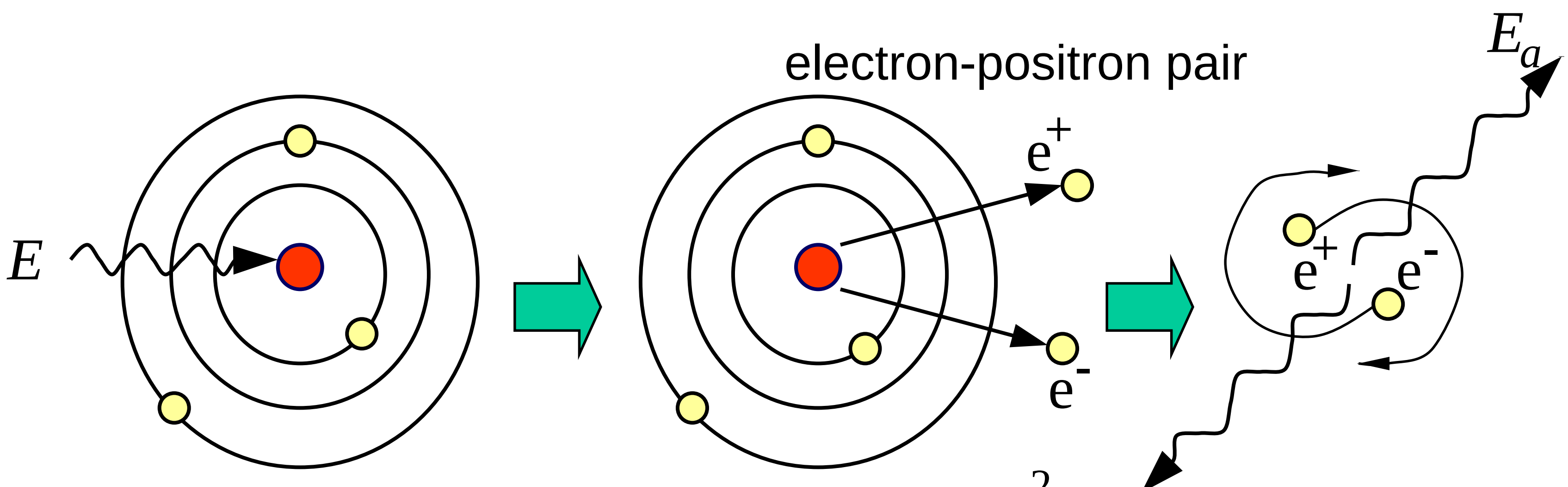
- Binding effects cause scattering to diverge into forms:
  - ***Incoherent scatter*** - scattering from individual electrons influenced by atomic binding; factors are used to correct the free-electron scattering model for these binding effects
  - ***Coherent scatter*** - Thomson scatter from all atomic electrons collectively - called Rayleigh scatter.

Coherent Scatter in Lead



Pair Production

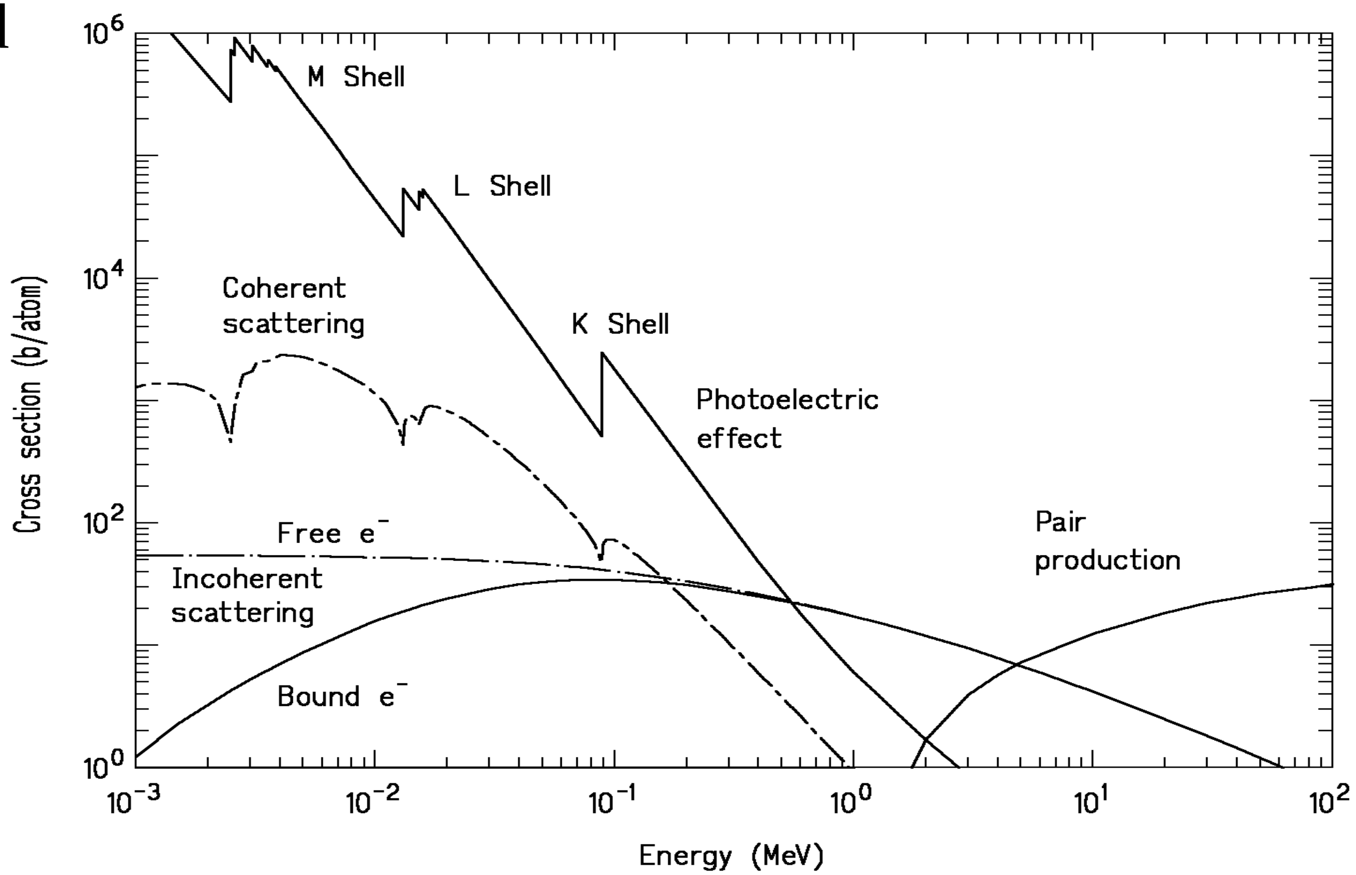
- Threshold reaction; photon energy > 1.022 MeV



$$T_+ + T_- = E - 2m_e c^2$$

Microscopic Cross Sections

Lead



Special Cases

- At high photon energies some other interactions are possible
- These include
  - Triplet production
  - Photo-nuclear interactions

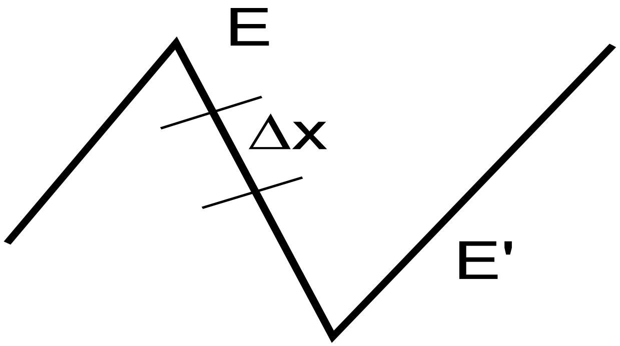
Triplet Production

- Charge must be conserved in any interaction, so how can we produce a triplet of charged particles?
- The answer is that pair production occurs in the vicinity of a nucleus but triplet production occurs in the vicinity of an electron
  - In this case, an electron positron pair is produced and the electron is ejected with kinetic energy, effectively producing two electrons and a positron
  - Often, the pair production cross section accounts for this, which is also pair production but in the presence of an electron

Photo-nuclear Reactions

- A gamma ray can sometimes interact with a nucleus
- Happens rarely because there are many more electrons in matter than nuclei
- Typical reaction is the (γ,n) reaction in which a gamma ray interacts by knocking a neutron out of the nucleus
- Becomes important at high gamma-ray energies

Photon Interaction Coefficients



Let $P(\Delta x)$ = probability of interaction in Δx

Then $\mu \equiv \lim_{\Delta x \to 0} \dfrac{P(\Delta x)}{\Delta x} =$ constant for given radiation and material

Compounds and Mixtures

$$\mu = \sum_i \mu_i = \sum_i N_i \sigma_i$$

$$\frac{\mu}{\rho} = \sum_i \frac{\mu_i}{\rho} = \sum_i \frac{\rho_i}{\rho}\frac{\mu_i}{\rho_i} = \sum_i w_i \left(\frac{\mu}{\rho}\right)_i$$

$w_i =$ weight (mass) fraction *i*th component

$\mu \equiv$ linear interaction coefficient

$=$ macroscopic cross section Σ (cm^{-1})

$\frac{\mu}{\rho} =$ mass interaction coefficient (cm^2/g)

Photon Cross Section Glossary

Interaction coefficient:
$\mu = \mu_{tot-coh} = \mu_c + \mu_{ph} + \mu_{pp}$

Energy transfer coefficient:
$\mu_{tr} = f_c\mu_c + f_{ph}\mu_{ph} + f_{pp}\mu_{pp}$

Energy absorption coefficient:
$\mu_{en} = f_c\left(1 - G_c\right)\mu_c + f_{ph}\left(1 - G_{ph}\right)\mu_{ph} + f_{pp}\left(1 - G_{pp}\right)\mu_{pp}$

f is the fraction of the photon energy that is deposited as kinetic energy (KE) of charged particles and G is the fraction of the KE of charged particles that is lost through radiative processes

Photon Cross Section Examples

water

E (MeV)	μ/ρ	μ_a/ρ	μ_{tr}/ρ	μ_{en}/ρ
0.01	4.87	4.79	4.79	4.79
0.1	0.165	0.0256	0.0256	0.0256
1	0.0707	0.0311	0.0311	0.0309
10	0.0221	0.0168	0.0162	0.0157

lead

E (MeV)	μ/ρ	μ_a/ρ	μ_{tr}/ρ	μ_{en}/ρ
0.01	132	132	131	131
0.1	5.62	5.51	2.28	2.28
1	0.0689	0.0407	0.0396	0.0397
10	0.0496	0.0457	0.0419	0.0310

Data Resources

- There are many government and international resources for nuclear data and cross sections
- Many of these are supported by the US Government or by Consortia
- We will describe some of these here that apply to photons

Some Organizations

- USA
  - NNDC National Nuclear Data Center, Brookhaven National Laboratory, Upton, NY
  - NIST National Institute of Standards and Technology, Gaithersburg, MD
  - RSICC Radiation Safety Information Computational Center, Oak, Ridge, TN
- International
  - NEA Nuclear Energy Agency Data Bank, Issy-les-Moulineaux, France;
  - IAEA International Atomic Energy Agency, Vienna, Austria

National Nuclear Data Center,
Brookhaven National Laboratory,
Upton, NY 11973-5000

http://www.nndc.bnl.gov/

Evaluated nuclear structure data file (ENSDF)
Levels adopted from ENSDF
Gammas adopted from ENSDF
Links to important data (e.g., NIST’s XCOM)

Physics Laboratory
Physical Reference Data
NIST National Institute of Standards and Technology

http://physics.nist.gov/PhysRefData/contents.html

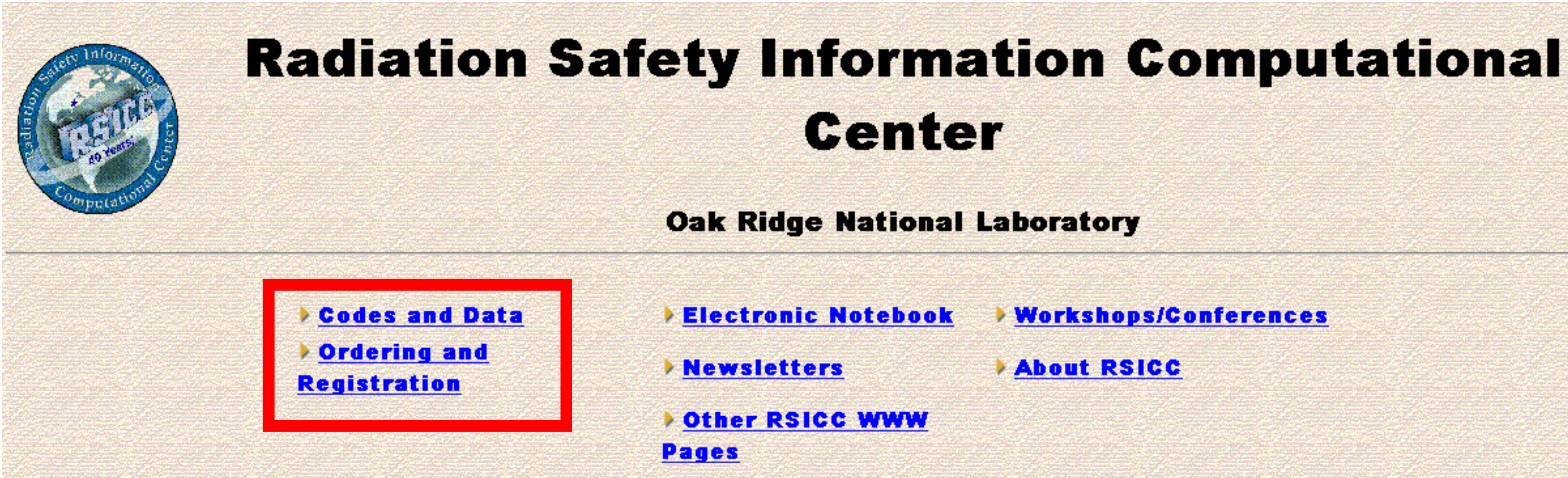
X-Ray and Gamma-Ray Data

- Note on the X-Ray Attenuation Databases
- X-Ray Attenuation and Absorption for Materials of Dosimetric Interest
- XCOM: Photon Cross Sections Database
- Bibliography of Photon Attenuation Measurements
- X-Ray Form Factor, Attenuation and Scattering Tables

Can download XCOM from here

If you run XCOM here, can get graphs and fancy formatted tables of photon cross sections

Can download tables of μ_{en} for elements and compounds and mixtures



http://www-rsicc.ornl.gov/rsicc.html

RSICC is a Specialized Information Analysis Center (SIAC) authorized to collect, analyze, maintain, and distribute computer software and data sets in the areas of radiation transport and safety. For photons

♦ DLC139: SIGMA-A: Photon Interaction and Absorption Cross Sections