# The Interaction of Radiation with Matter: Charged Particle and Electron Interactions

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# Outline

# Energy loss processes for ions in matter

In condensed matter, photons and neutrons interact discretely but charged particles continuously interact with electrons and protons in the nucleus via the long-range Coulomb force. Most interactions however are elastic (Rutherford) scattering with atomic electrons:

- Interactions of heavy ions with atomic electrons are characterized by very small exchanges of energy.
- The maximum energy loss per interaction, $\Delta E = 4Em_e/M$, occurs when all of the relative momentum of the electron to the charge particle changes sign.
- For a proton $\Delta E/E \approx 1/450$
  For an $\alpha$ particle $\Delta E/E \approx 1/1800$
  Ions undergo 1000's of interactions before coming to a stop

# The Bethe-Bloch formula

The linear stopping power, S=-d$E$/d$x$, is a measure of the average rate of energy loss along the trajectory (not necessarily straight) of a charged particle.

$$S = \frac{1}{(4\pi\epsilon_0)^2}\frac{4\pi e^4 z^2}{m_e \nu^2} NZ \left[\ln\frac{2m_e\nu^2}{I} - \ln(1-\beta^2) - \beta^2\right] \qquad (1)$$

- where $\nu = \sqrt{2E/M}$ is the velocity of the charge, $z$, particle
- $N$ is the number of atoms per unit volume, $Z$ is the number of electrons per atom.
- $I$ is the average ionization/excitation potential of the medium. $I$ is essentially an empirical item but can be derived from elastic (Rutherford) scattering by atomic electrons in the Born approximation - all electrons are considered to be dielectric oscillators at rest
- I for hydrogen is 20.4 eV
- For heavier media $I \approx 11Z$ eV or $9.1(Z + 1.9Z^{1/3})$ eV

# Stopping powers for ions

The stopping power for ions in a particular medium is:

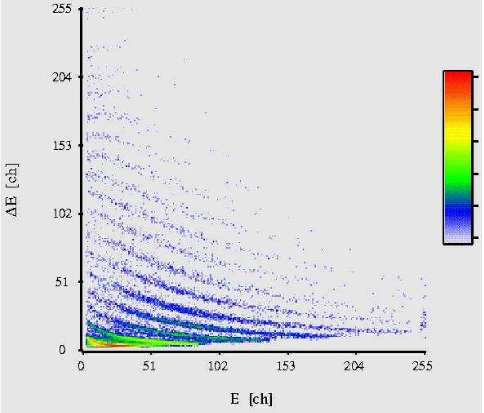
$$S \propto \frac{z^2}{\nu^2} \log\left[f\left(\nu^2\right)\right] \tag{2}$$

- $S$ is independent of the mass of the ions of the same element travelling at the same velocity, i.e. a 2 MeV deuteron has the same $S$ as a 1 MeV proton whereas a 4 MeV $\alpha$-particle has $4 \times S$
- The $\Delta E.E$ telescope is used in nuclear physics to distinguish particle types:

  $$\Delta E.E = S\Delta x.1/2A\nu^2 \propto z^2 A \log\left[f\left(\nu^2\right)\right] \tag{3}$$

  where $\Delta x$ is the thickness of a thin detector (e.g. a semiconductor). The log term is a slowly varying function of the particle velocity.

# Typical data from a $\Delta E.E$ telescope



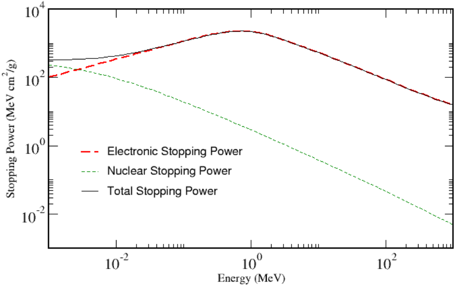
# Failure of Bethe-Bloch approximation

The Bethe-Bloch model becomes invalid at low energies:

1. When the velocity of the charged particle is comparable with K shell electron (10 MeV/$A$)
2. Charge exchange - the ion picks up electrons from the medium reducing its charge. Significant below $E \approx 0.2$ MeV/$A$
3. Nuclear interactions dominate below 0.01 MeV

# Tabulated Data

The Mass-Stopping Power, $S/\rho \propto Z/A$, with the exception of H, is slowly varying with Z. $\alpha$ particles in water



`http://physics.nist.gov/PhysRefData/Star/Text/contents.html`

# Ranges of ions

Range, $R$, is determined by integrating using $\mathrm{d}E = 1/2A\ \mathrm{d}\nu^2$:

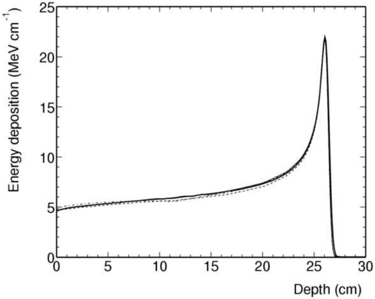
$$\begin{aligned} S &= -\frac{\mathrm{d}E}{\mathrm{d}x} \\ R &= \int_0^E \frac{\mathrm{d}E}{S} \\ R &= A/z^2 \int_0^\nu \frac{\nu^2 \mathrm{d}\nu^2}{\ln\left[f\left(\nu^2\right)\right]} \end{aligned} \tag{4}$$

The range of ions *is* dependent the mass number, $A$, e.g.

- 20 MeV deuteron has twice the range of 10 MeV proton
- a 40 MeV alpha has the same range (about 1 mm)

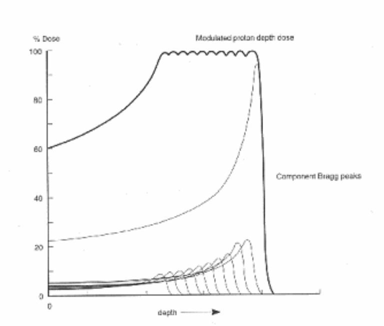
# Energy deposition

Most energy is deposited at the end of the track, i.e., in the Bragg peak



Monte Carlo calculations of dose versus depth for 200 MeV proton beams using GEANT4 and SHIELD-HIT `http://www.hmi.de/isl/att`

# Dose distributions from Proton Therapy Beams



In medicine, proton and ion beams are used to precisely deliver radiation to deep-seated tumors `http://www.hmi.de/isl/att/att-1_en.html`

# Ion Beam Therapy

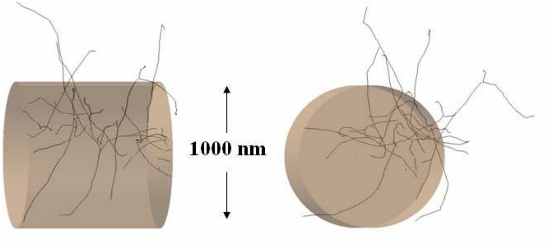
`http://www.wordsun.com/boc589.html`

# Fast electrons in matter

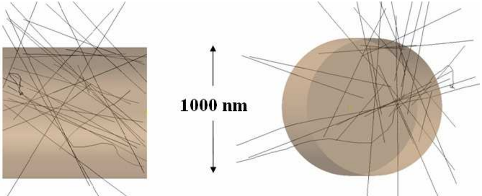
Electrons lose $E$ by collisions with atomic $e^-$'s. Take 100's of collisions to slow down completely. Three main differences to ions:

1. With electron-electron inelastic scattering (known as M oller scattering, where atomic binding effects are ignored), the projectile and target are indistinguishable. The maximum energy energy lost is $E/2$. 'Secondary' electrons can also travel large distances.
2. Electrons are often relativistic - $m_e c^2 = 0.511$ keV (the rest mass) - this needs to be considered if kinetic energy is comparable to or larger than rest mass.
3. Because the masses of projectile and target are identical they can scatter in any direction and lose large fractions of energy. The path of an electron is very erratic.

# Electron tracks



Electron tracks from Mo/Mo X-rays used in mammography



Electron tracks from Sr/Y-90: a 0.5 and 2 MeV beta

# Collision stopping-power

Collisional loses, $S_c$ are described by a modified version of the Bethe-Bloch formula.

$$S_c = \frac{1}{(4\pi\epsilon_0)^2}\frac{2\pi e^4 NZ}{m_e \nu^2}\left(\ln\frac{m_e\nu^2 E}{2I^2(1-\beta^2)} - g(\beta^2)\right) \tag{5}$$

where $g(\beta^2)$ is the density effect correction, a function of the electron velocity

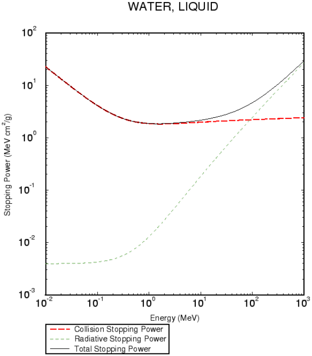
- $g(\beta^2) = -(\ln 2 + 1)\beta^2$ for high energies
- $g(\beta^2) = \ln 2 + 1$ for low energies

# Radiative stopping power

Energy is also lost through Bremsstrahlung - braking radiation - which occurs when an $e^-/e^+$ passes very close to a nucleus.

$$S_r = \frac{NEZ(Z+1)e^4}{137 m_e^2 c^4}\left(4\ln\frac{2E}{m_e c^2} - \frac{4}{3}\right) \tag{6}$$

The total stopping power, $S = S_c + S_r$

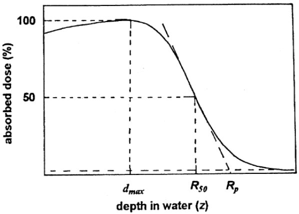


# Straggling

The $e^-$ range is broadly distributed due to straggling:

1. of the range
2. of the energy distribution

The range can't be calculated from the Bethe-Bloch formula because the trajectory of the $e^-$ not a straight-line. A detour factor which is derived from the scattering power of the medium or experimentally is used.



- The limited range of high energy electrons makes them useful for treating superficial tumors overlying sensitive organs

# Cerenkov radiation

Electrons are often relativistic so produce Cerenkov radiation in optically dense media

- An EM shock-wave produced when charged particles travel faster than light propagates in the medium, $\nu = c/n$, where $n$ is the refractive index of the medium.
- A bluish light is emitted in a cone at $\cos\theta < 1/\beta n$

Energy lost is small but can be used as mode of detection (Cerenkov detector) and can contaminate a signal in optically based detection system (e.g. optical fibres)